Journal of Statistical Physics, Vol. 34, Nos. 5/6, 1984

Spin Glasses

A. P. Young[1]

Received September 30, 1983

Recent developments in the theory of spin glasses are discussed. There has been considerable progress, due to Parisi, Sompolinsky, and others, towards understanding the infinite range (mean field) model of Sherrington and Kirkpatrick. Relaxation times diverge in the thermodynamic limit, and this nonergodic behavior is now understood to be the cause of earlier difficulties. There has been less progress in the study of more realistic models with short-range interactions, but numerical studies have shown rather clearly the absence of a finite temperature transition in two dimensions. There is probably no transition in $d = 3$ either, though the evidence is less clearcut, which makes it difficult to understand the sharpness of the "freezing" observed experimentally. Well below the freezing temperature ESR and torque measurements have been fairly well explained by a theory of Henley, Sompolinsky, and Halperin, in which an important ingredient is anisotropy due to the Dzyloshinsky–Moriya interaction proposed by Fert and Levy.

KEY WORDS: Spin glasses; phase transition; metastable states; replicas; computer simulations; anisotropy.

Interest in spin glasses continues unabated, and several hundred more papers on this topic have appeared since the previous STATPHYS conference three years ago. All that can be accomplished in a short review like this is to give a general impression of how the field has developed. We shall see that progress can be reported in certain areas, but many questions remain unanswered. More complete references to other work are given in the reviews by Fischer[(1)] and Rammal and Souletie.[(2)]

For a theorist a spin glass model must have both frustration[(3)] and disorder. Systems with frustration cannot minimize simultaneously the

[1] Department of Mathematics, Imperial College, London SW7 2BZ, U.K.

0022-4715/84/0300-0871$03.50/0

energy of each term in the Hamiltonian because of competition between different requirements. A simple example of a frustrated system is an antiferromagnet on a triangular lattice. This is not a spin glass, however, because it has no disorder.[4] Such properties can arise in experimental systems in various ways. The canonical spin glasses,[5] such as CuMn and AuFe, consist of dilute randomly placed magnetic atoms in a nonmagnetic host metal interacting with an RKKY interaction, which of course changes sign with distance and gives the frustration. Insulating materials can also show spin glass behavior, for instance $Eu_xSr_{1-x}S$, where frustration is due to competition between first and second neighbor interactions of different sign,[6] and real amorphous glasses with magnetic atoms[7] where differences in bond angles can alter the sign of the superexchange interaction.

Experimentally, one of the most striking signatures of spin glass behavior is a fairly sharp cusp[8] in the low field ac susceptibility. Below the cusp, or freezing, temperature T_c, there are strong irreversible effects showing that enormous relaxation times occur. For some materials, such as $Eu_xSr_{1-x}S$, a fairly large[9] dependence of T_f on the logarithm of the measuring frequency, ν, is observed, whereas other systems, such as CuMn, show very little[10] variation with frequency. A more direct indication of a phase transition, though the experiment is a bit harder, is to look at the dc nonlinear susceptibility, χ_{nl}, defined to be the coefficient of h^3 in the magnetization, i.e.,

$$m = \chi h - \chi_{nl} h^3 + \cdots \tag{1}$$

As we shall see, theory predicts that χ_{nl} diverges if there is a spin glass temperature, and experiments[11] show that it indeed becomes very large in a way consistent with a power law divergence. The advantage of these experiments is that they are above T_f and so should not be affected by hysteresis effects. It should be pointed out, however, that even a very small error in m due to a *small* irreversible component which could be present even just above T_f can give a large error in χ_{nl}. Other experiments that will be mentioned are those[12,13] which plot out the freezing temperature (defined in different ways for different experiments) against h and find evidence for a "transition line" $T_c(h)$, where, for small h,

$$T_c(h) - T_c(0) \propto h^{\lambda} \tag{2}$$

where λ is close to 2/3. As will be pointed out later, mean field theory (MFT) also predicts this behavior, including the value of the exponent.

Having given a very brief and personal view of the experimental situation, let us now give a somewhat fuller account of the theory. Most theoretical work falls naturally into one of three categories:

(i) Study of the MFT.

(ii) Study of more realistic models with short-range interactions in three (and two) dimensions. The object is to determine whether these models have a transition as in MFT. If not, the predictions of MFT should be quite inappropriate and the observed effects must be due to a gradual, though fairly rapid, freezing out of the spins when the temperature is lowered and the time scale of observation is fixed.

(iii) Investigation of properties well below T_f, where the system is certainly frozen on any experimental time scale and the object is to determine the nature of the anisotropy which plays an important role in, for instance, ESR and torque measurements.

Let us start with MFT, which is the category with the most progress to report. Like much of the theory, it is based on a model proposed by Edwards and Anderson[14] where one considers a regular lattice with spins at each point and interactions between them which are independent random variables, with a distribution which usually depends on the distance between the sites. For instance, one could have nearest-neighbor coupling only and this will be discussed later. Sherrington and Kirkpatrick[15] (SK) proposed, by analogy with ferromagnetism, where MFT is exact in the infinite range limit, to define MFT for spin glasses as the exact solution of an infinite range Edwards–Anderson model, where the distribution is independent of distance. This unphysical assumption should make the model solvable. The geometry of the lattice is now irrelevant so there is no space dimension associated with the model, but it probably corresponds most closely to a short-range model in the limit of infinite space dimension.

The Hamiltonian is given by

$$H = -\sum_{\langle i,j\rangle} J_{ij} S_i S_j - h \sum_i S_i \tag{3}$$

where h is a uniform field and the J_{ij} have a Gaussian distribution whose mean, J_0/N, and variance, $1/N$, both have to scale as N^{-1}, where N is the number of spins, to obtain a sensible but nontrivial thermodynamic limit. Only a symmetric distribution will be considered here. We have written Eq. (3) for Ising spins, $S_i = \pm 1$, and will only discuss this case, but the Heisenberg model has also been investigated.[16]

In SK's original solution a transition occurs in zero field at $T_c = 1$ in our units and the low-temperature phase is characterized by a single order parameter, q, defined by

$$q = \langle\langle S_i \rangle_T^2 \rangle_J \tag{4}$$

where $\langle \cdots \rangle_T$ denotes a statistical mechanics average for a given set of interactions and $\langle \cdots \rangle_J$ is an average over the interactions. This solution was subsequently shown by Almeida and Thouless[17] (AT) to be unstable

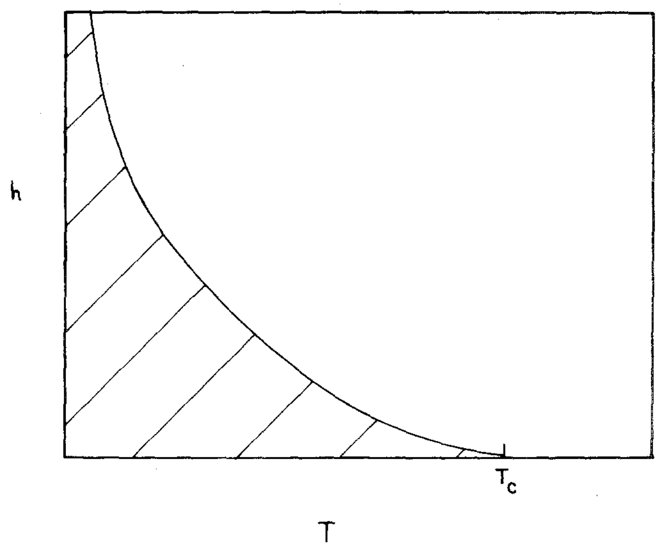

Fig. 1. The shaded region is where the Sherrington Kirkpatrick solution for the infinite range Ising spin glass is unstable and "replica symmetry breaking," i.e., nonergodic behavior occurs.

below a line in the $h-T$ plane terminating at $h = 0$, $T = T_c$, see Fig. 1, and for small h varies as in Eq. (2). Below the AT line a single order parameter description is no longer correct. Both the SK paper and the subsequent calculations used the replica trick where the identity

$$\langle \ln Z \rangle_J = \lim_{n \to 0} \frac{\langle Z^n \rangle_J - 1}{n} \tag{5}$$

is used to perform the quenched average and obtain a nondisordered Hamiltonian. Labeling the n replies by α, β etc., the order parameter, q, in Eq. (4) becomes

$$q = q^{\alpha\beta} = \langle S_i^\alpha S_i^\beta \rangle \tag{6}$$

for any $\alpha \neq \beta$ in the replica formulation. Below the AT line it is necessary to break replica symmetry, i.e., to make $q^{\alpha\beta}$ depend on α and β, whereas the SK solution had them all equal. The most successful such scheme is due to Parisi,[(18)] who breaks up the matrix $q^{\alpha\beta}$ into a fractal structure, and finally, after taking the limit $n \to 0$, reduces the matrix to a continuous function $q(x)$ in the interval from 0 to 1. The predicted form for $q(x)$ is sketched in Fig. 2 for $h \to 0$, $T < T_c$.

It was initially unclear what is the significance of this variable x and, in particular, how $q(x)$ is related to physically meaningful quantities such

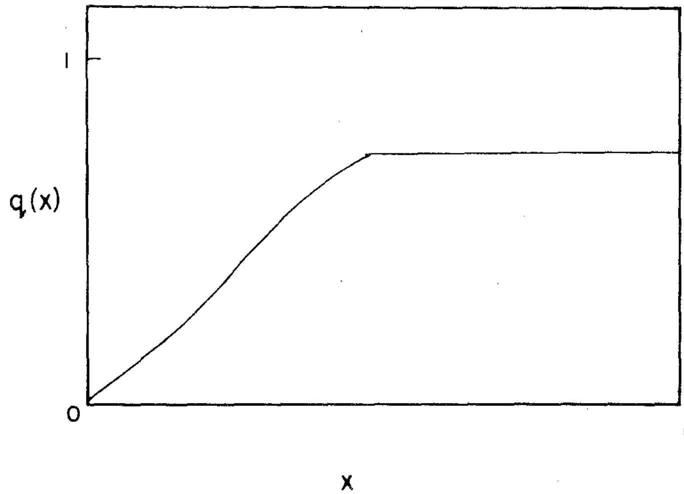

Fig. 2. A qualitative sketch of Parisi's order parameter function $q(x)$ for $T < T_c$, $h \to 0$.

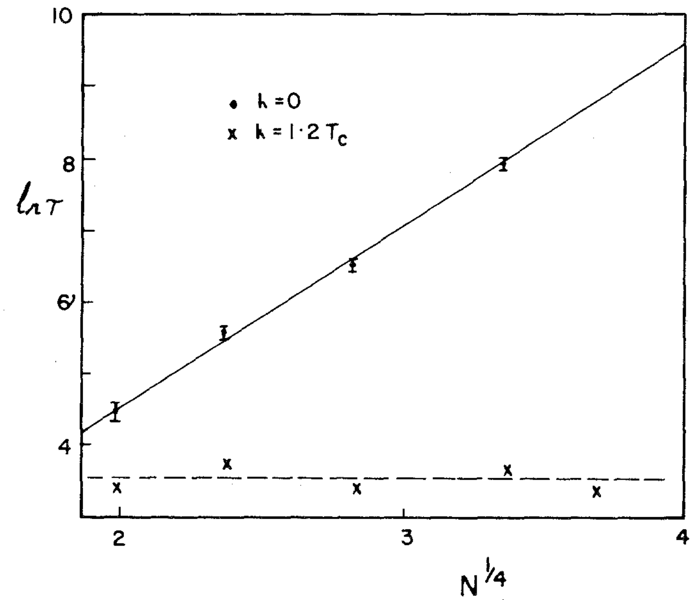

Fig. 3. τ is the longest relaxation time for the SK model and N the number of spins. The temperature is $0.4T_c$ and data for $h = 0$ and $h = 1.2T_c$ are shown. At $h = 0$ $\ln \tau \propto N^{1/4}$ which shows nonergodic behavior, while at $h = 1.2T_c$, which is above the AT line, the relaxation times are independent of system size.

as q in Eq. (4). The characteristics of the phase below the AT line were clarified by Sompolinsky,[19] who showed that there are relaxation times which diverge when $N \to \infty$. Subsequently, direct numerical evidence for this was obtained,[20] see Fig. 3. This idea is connected with the fact that the mean field equations of Thouless, Anderson, and Palmer[21] (TAP), which describe the magnetizations for a particular set of J_{ij}, have many solutions.[22] One expects that these (or perhaps a subset) describe the minima in phase space which, to be stable at finite temperatures, must be separated from other minima by a barrier whose height diverges when $N \to \infty$. Sompolinsky's divergent time scales presumably correspond to the rare fluctuations over these barriers which can occur for finite N.

With this picture it is clear that many order parameters can be defined. For instance, let us consider one of these minima (the words "phases," "valley," and "solutions" will be used interchangeably) and define an order parameter for that valley, q^{ss}, by

$$q^{ss} = \frac{1}{N} \sum_i (m_i^s)^2 \tag{7}$$

where m_i^s is the magnetization of site i in phase s. Really we would like to avoid having to specify which phase we are talking about and a statistical description may be obtained by noting that statistical mechanics sums over them all with a weight $P(s)$, where[23]

$$P(s) = \frac{1}{Z} \exp(-\beta F_s) \tag{8}$$

One can therefore define the Edwards–Anderson order parameter, q_{EA}, by

$$q_{\text{EA}} = \left\langle \sum_s P(s) q^{ss} \right\rangle_J \tag{9}$$

which is presumably the same as q^{ss} for one solution of minimum free energy. Alternatively, the statistical mechanics order parameter q, of Eq. (4), is given by

$$q = \left\langle \sum_{s,s'} P(s) P(s') q^{ss'} \right\rangle_J \tag{10}$$

where

$$q^{ss'} = \frac{1}{N} \sum_i m_i^s m_i^{s'} \tag{11}$$

measures the overlap between magnetizations in phases s and s'.

de Dominicis and Young,[23] using the permutation symmetry of the replica Hamiltonian, have argued that

$$q = \lim_{n \to 0} \frac{1}{n(n-1)} \sum_{\alpha \neq \beta} q^{\alpha\beta} = \int_0^1 q(x)\, dx \tag{12}$$

where the first equality is true in general and the second is for Parisi's ansatz. It follows almost trivially, comparing Eqs. (10) and (12), that[24,25]

$$\frac{dx(q')}{dq'} = \left\langle \sum_{s,s'} P(s)P(s')\delta(q' - q^{ss'}) \right\rangle_J \tag{13}$$

which is a probability distribution for the overlap between magnetizations of the solutions to equal q'. This provides the physical significance of Parisi's replica symmetry breaking scheme.

Many of the detailed predictions of Sompolinsky's[19] dynamical theory agree with results using the above interpretation of replicas[23,25,26] except that he finds $q = q\ (x = 0)$ instead of Eq. (10). The reason for the discrepancy is not clear, but it has been proposed[27] that the dynamical approach may describe a nonequilibrium situation.

Recently[28] the right-hand side of Eq. (13) has been evaluated by computer simulations, and the results are consistent with the predictions for the function $q(x)$, see Fig. 2, modified by fairly substantial finite size corrections. Thus the MFT is certainly understood qualitatively in terms of many phases with infinite barriers between them, and on a quantitative level Parisi's theory is close to, and perhaps equal to, the exact solution.

Next we discuss the second category of theories where short-range (and hence more realistic) models are studied. One important motivation for this is to determine the lower critical dimension, d_L, for the spin glass transition. This is the space dimension below which fluctuations, neglected in MFT, destroy the transition which MFT predicts. If $d_L > 3$ then real systems do not have a sharp transition and the experiments have to be due to a gradual freezing of the spins on experimental time scales.

One of the first attempts to determine d_L was due to Fisch and Harris.[29] They calculated 10 terms in the high-temperature series for χ_{SG}, defined by

$$\chi_{SG} = \frac{1}{N} \sum_{i,j} \langle \langle S_i S_j \rangle_T^2 \rangle_J \tag{14}$$

for the nearest-neighbor symmetry Edwards–Anderson model with Ising spins on cubic lattices of dimension d. Clearly, χ_{SG} diverges when the correlations become very long range (but of random sign) and this signals a transition. For the model considered $\chi_{SG} = 3T^3\chi_{nl}$. An analysis of the series gives no divergence below $d = 4$, implying $d_L = 4$, and similar results were subsequently obtained[30] for Heisenberg spins. However, a reanalysis of the $d = 3$ Ising series[31] showed that the situation is not completely clearcut, some methods of analyzing the series giving a transition, others not. Clearly it would be useful if several more terms in the series could be obtained.

For Ising spins one can evaluate the partition function exactly for small systems and Morgenstern and Binder[32] have succeeded in doing this for the Edwards–Anderson model for sizes up to 18×18 in $d = 2$ and $4 \times 4 \times 10$ in $d = 3$. They calculated

$$\Gamma(R_{ij}) = \left\langle \langle S_i S_j \rangle_T^2 \right\rangle_J \tag{15}$$

where R_{ij} is the distance between sites i and j, and found that it always decayed exponentially, showing the absence of a transition. This is very convincing in $d = 2$ where finite size effects only play a role at rather low temperature, but the linear dimensions in $d = 3$ were so small that one probably cannot draw definite conclusions. In $d = 2$ it was found that χ_{SG} and the correlation length ξ appear to have power law divergences as $T \to 0$, specifically

$$\chi_{SG} \sim T^{-4}, \qquad \xi \sim T^{-2} \tag{16}$$

One can go to much larger sizes with Monte Carlo simulation but there another problem appears. At low temperatures relaxation times increase very rapidly so the system cannot be brought to equilibrium. Nonetheless, simulations have been carried out[33] which give results consistent with Eq. (16) over an intermediate temperature range and, in addition, give fairly precise results for dynamics. More quantitatively, if we define

$$q(t) = \frac{1}{N} \sum_i \langle\langle S_i(t_0) S_i(t_0 + t) \rangle_T \rangle_J \tag{17}$$

where t_0 is an equilibration time, and extract an average relaxation time, τ, from $\tau = \int q(t)\,dt$ and characteristic energy barrier ΔE from $\Delta E = T \log \tau$ then the simulations find[33]

$$\Delta E = a + b/T \tag{18}$$

in $d = 2$. Consequently, energy barriers are always finite, except at $T = 0$, but increase as $T \to 0$ because of increasing correlations between the spins, as described by Eq. (16).

One can also calculate τ in the presence of a uniform magnetic field, h, and plot lines of constant τ in the h–T plane. Data for the $d = 2$ simulations is shown in Fig. 4 and at low temperatures varies approximately as $T(h) - T(h = 0) \propto h^{2/3}$, just like the AT line. However, this is probably a coincidence since the physics is very different. The AT line is a sharp transition while the data in Fig. 4 are a purely dynamical effect. Similar numerical results have also been obtained by Kinzel and Binder.[34] The data in Fig. 4 look very like results[13] on $Eu_xSr_{t-x}S$, which suggests that the gradual freezing picture may be appropriate for this insulating material.

Some analogous simulations have also been carried out on a simple cubic lattice.[35] Relaxation times and the spin glass susceptibility increase

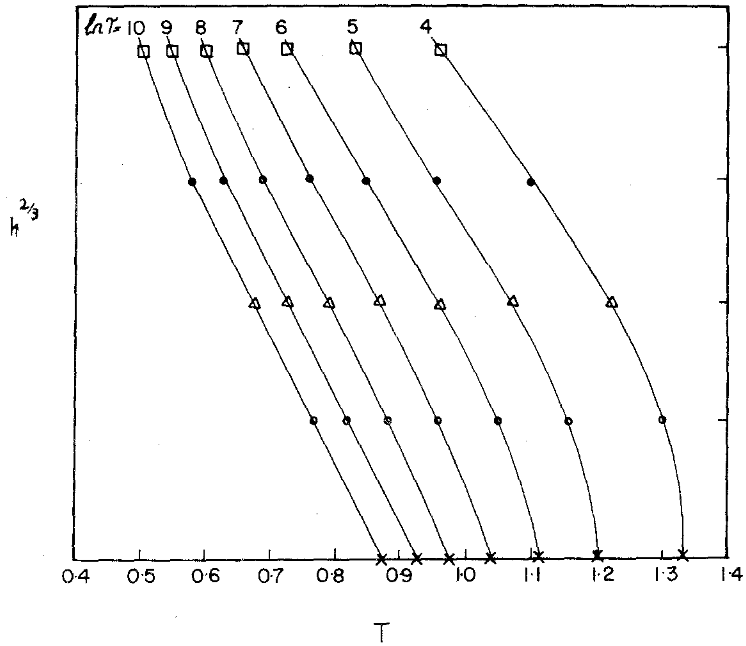

Fig. 4. Lines of constant average relaxation time τ, are plotted in the $h^{2/3}$–T plane for $d=2$, $\pm J$ nearest-neighbor Ising spin glass. The field values are $h=0$, 0.1, 0.2, 0.45, and 0.7. At low temperatures these lines have a similar behavior to the AT line, $\delta T \propto h^{2/3}$, but the data here show a purely dynamical effect, with a distinct line for each time scale.

faster than in $d=2$, and if a transition occurs it happens at rather low temperatures $T \lesssim 1.3$, which is significantly below the lowest temperature at which equilibration can be attained. However, the data are also consistent with a transition only at $T=0$.

By generalizing their earlier work on mean field dynamics[19,36] to short-range models, Sompolinsky and Zippelius[37] have found that $d_L = 4$. This is the most convincing demonstration to date that fluctuations wipe out the spin glass transition in $d=3$. However, one then has the severe problem of understanding experiments on metallic systems such as CuMn which apparently do show a transition. The data for χ_{nl}[11] seem particularly compelling. In the last three years, then, theorists have become more than ever convinced that $d_L = 4$ and experimentalists more than ever sure that a transition occurs in $d=3$. Hence this paradox is no nearer being resolved. Could it be that RKKY or Dzyloshinski–Moriya[38] interactions are really long range and therefore change d_L for metallic systems?

Finally, we report briefly on the third class of theories, which is concerned with behavior well below the freezing temperature, where the spins are certainly trapped in some metastable state for the duration of the experiment. The largest transitions are the isotropic Heisenberg exchange forces while anisotropy, which prevents free overall rotation of the spins, is relatively weaker. However, torque[39] and ESR[40] measurements probe directly this anisotropy. Fert and Levy[38] have argued that Dzyloshinski–Moriya interactions are the largest source of anisotropy in canonical spin glasses, such as CuMn. This gives[39,41] a unidirectional anisotropy, i.e., the free energy changes by F for rotation through an angle ϑ, where

$$F = K_1 \cos\vartheta \tag{19}$$

The more conventional uniaxial anisotropy gives a $\cos 2\vartheta$ dependence. The change in free energy should only depend on ϑ, the magnitude of the rotation, and not on the direction about which the rotation takes place. This, and the $\cos\vartheta$ variation, have been observed.[39]

A detailed theory for ESR frequencies, incorporating this anisotropy, has been developed by Henley *et al.*[41] They note that one needs to follow a triad of three orthogonal vectors to describe the rotation of a spin glass, not just the direction of a single vector which is sufficient for collinear magnets. As a result, there are three resonance modes, and these have apparently all been seen.[40] If large angles of rotation are induced, for instance by applying a field at a large angle to the field in which the sample is cooled, the theory works less well and the assumption that the spins rotate rigidly seems to be no longer correct. Nonetheless, many of the important features of the ESR data are explained by the triad model with Dzyloshinski–Moriya anistropy.

To conclude, there has certainly been progress in the last three years, but crucial questions concerning the behavior of spin glasses near the freezing temperature remain unanswered.

REFERENCES

1. K. Fischer, *Phys. Stat Sol. (b)* **116**:357 (1983).
2. R. Rammal and J. Souletie, in *Magnetism of Metals and Alloys*, M. Cyrot, ed. (North-Holland, Amsterdam, 1982).
3. G. Toulouse, *Commun. Phys.* **2**:115 (1977).
4. A recent discussion of "what is a spin glass" is given by D. Sherrington, in *Proceedings of Heidelberg Colloquium on Spin Glasses* (Springer, Berlin, to be published).
5. See, for instance, the discussion in Ref. 2.
6. H. Maletta, and W. Felsch, *Phys. Rev. B* **20**:1245 (1979).
7. L. E. Wenger, C. A. M. Mulder, A. J. van Duyneveldt, and M. Hardiman, *Phys. Lett.* **87A**:439 (1982).
8. V. Cannella and J. Mydosh, *Phys. Rev. B* **6**:4220 (1972).
9. J. Ferré, J. Rajchenbach, and H. Maletta, *J. Appl. Phys.* **52**:1697 (1981).
10. C. A. M. Mulder, A. J. van Duyneveldt, and J. Mydosh, *Phys. Rev. B* **25**:515 (1982); E. D. Dahlberg, M. Hardiman, R. Orbach, and J. Souletie, *Phys. Rev. Lett.* **42**:401 (1979).

11. R. Omari, J. J. Préjean, and J. Souletie (to be published); P. Monod and H. Bouchiat, *J. Phys. (Paris) Lett.* **43**:405 (1982); B. Barbara, A. P. Malozemoff, and Y. Imry, *Phys. Rev. Lett.* **47**:1852 (1981).
12. R. V. Chamberlin, M. Hardiman, L. A. Turkevich, and R. Orbach, *Phys. Rev. B* **25**:6720 (1982); Y. Yeshurun, L. J. P. Ketelsen, and M. B. Salamon, *Phys. Rev. B* **26**:1491 (1982).
13. N. Bontemps, J. Rajchenbach, and R. Orbach, *J. Phys. (Paris) Lett.* **44**:L47 (1983); N. Bontemps, in *Proceedings of Heidelberg Colloquium on Spin Glasses* (Springer, Berlin, to be published); H. Maletta, *ibid.*
14. S. F. Edwards and P. W. Anderson, *J. Phys. F* **5**:965 (1975).
15. D. Sherrington and S. Kirkpatrick, *Phys. Rev. Lett.* **35**:1792 (1975) (referred to as SK).
16. M. Gabay and G. Toulouse, *Phys. Rev. Lett.* **47**:201 (1981); D. M. Cragg, D. Sherrington, and M. Gabay, *Phys. Rev. Lett.* **49**:158 (1982).
17. J. R. de Almeida and D. J. Thouless, *J. Phys. A* **11**:983 (1978).
18. G. Parisi, *J. Phys. A* **13**:1101, L115, 1887 (1980).
19. H. Sompolinsky, *Phys. Rev. Lett.* **47**:935 (1981).
20. N. D. Mackenzie and A. P. Young, *Phys. Rev. Lett.* **49**:301 (1982); and to be published.
21. D. J. Thouless, P. W. Anderson, and R. G. Palmer, *Phil. Mag.* **35**:593 (1977) (referred to as TAP).
22. A. J. Bray and M. A. Moore, *J. Phys. C* **13**:L469, (1980); C. de Dominicis, M. Gabay, T. Garel, and P. Orland, *J. Phys. (Paris)* **41**:923 (1980); F. Tanaka and S. F. Edwards, *J. Phys. F* **10**:2769 (1980).
23. C. de Dominicis and A. P. Young, *J. Phys. A* **16**:2063 (1983).
24. G. Parisi, *Phys. Rev. Lett.* **50**:1946 (1983).
25. A. Houghton, S. Jain, and A. P. Young, *J. Phys. C* **16**:L375 (1983).
26. A. Houghton, S. Jain, and A. P. Young, *Phys. Rev. B* **28**:2630 (1983).
27. C. de Dominicis and A. P. Young, *J. Phys. C* **16**:L641 (1983).
28. A. P. Young (to be published).
29. R. Fisch and A. B. Harris, *Phys. Rev. Lett.* **38**:785 (1977).
30. P. Reed, *J. Phys. C* **11**:L979 (1978).
31. R. G. Palmer (unpublished).
32. I. Morgenstern and K. Binder, *Phys. Rev. Lett.* **43**:1615 (1979); *Phys. Rev. B* **22**:288 (1980); *Z. Phys. B* **39**:227 (1980).
33. A. P. Young, *Phys. Rev. Lett.* **50**:917 (1983).
34. W. Kinzel and K. Binder, *Phys. Rev. Lett.* **50**:1509 (1983).
35. A. P. Young, in *Proceedings of Heidelberg Colloquium on Spin Glasses* (Springer, Berlin, to be published); and to be published.
36. H. Sompolinsky and A. Zippelius, *Phys. Rev. Lett.* **47**:359 (1981); *Phys. Rev. B* **25**:6860 (1982).
37. A. Zippelius, in *Proceedings of Heidelberg Colloquium on Spin Glasses* (Springer, Berlin, to be published).
38. A. Fert and P. M. Levy, *Phys. Rev. Lett.* **44**:1538 (1980).
39. A. Fert and F. Hippert, *Phys. Rev. Lett.* **49**:1508 (1982).
40. S. Schultz, E. M. Gullikson, D. R. Fredkin, and M. Tovar, *Phys. Rev. Lett.* **45**:1508 (1980); E. M. Gullikson, D. R. Fredkin, and S. Schultz, *Phys. Rev. Lett.* **50**:537 (1983).
41. C. L. Henley, H. Sompolinsky, and B. I. Halperin, *Phys. Rev. B* **25**:5849 (1982).